Section 4.4, Problem 32: $\sqrt[3]{25} = (25)^{1/3} = (5^2)^{1/3} = 5^{2/3}$, so $\log_5 \sqrt[3]{25} = 2/3$.
Grading: 1 point.

Section 4.4, Problem 118(a,b,c):

(a) $H = -(0.724 \log 0.724 + 0.126 \log 0.126 + 0.009 \log 0.009 + 0.048 \log 0.048 + 0.002 \log 0.002 + 0.062 \log 0.062 + 0.029 \log 0.029) \approx 0.4125$

(b) There are 7 categories in this example, so $H_{max} = \log 7 \approx 0.8451$.

(c) $E_H = \dfrac{H}{H_{max}} \approx 0.4987$.

Comment: *The Shannon Diversity Index is an instance of Shannon's entropy function, which he defined in the context of information theory. In this case, it is a measure of the uncertainty of what the race is of a randomly selected person in the U.S. The bigger the value of H, the harder it is to 'guess' what the race will be. If there were only one racial group, then the proportion of that group in the population would be* 1 *and*

$$H = -1 \cdot \log 1 = 0,$$

i.e., there would be no uncertainty as to a randomly selected person's race.

Section 4.5, Problem 52: $\log \left[\dfrac{x^3\sqrt{x-1}}{(x-2)^2} \right] = 3\log x + \frac{1}{2}\log(x-1) - 2\log(x-2)$.

Section 4.7, Problem 42: The value of the investment after t years, assuming an initial investment of \$25000, and a 7% interest rate, compounded continuously is

$$V(t) = 25000e^{0.07t}.$$

If $V(t_1) = 80000$, then

$$80000 = 25000e^{0.07t_1} \;\Rightarrow\; e^{0.07t_1} = \frac{80000}{25000} = 3.2 \;\Rightarrow\; 0.07t_1 = \ln 3.2 \;\Rightarrow\; t_1 = \frac{\ln 3.2}{0.07} \approx 16.6.$$

I.e., it will take 16.6 years for the investment to grow to 80000.
Section 4.8, Problem 12: The proportion of carbon-14 left after t years is e^{kt}, where

$$k = \frac{1}{5730}\ln(1/2) \approx -0.000121,$$

is obtained from the assumption that the half-life of carbon-14 is 5730 years. If 70% of the fossilized leaf's carbon-14 remains and t is the age of the leaf (i.e., the number of years since it died), then

$$e^{-0.000121t} = 0.7 \;\Rightarrow\; -0.000121t = \ln 0.7 \;\Rightarrow\; t = \frac{\ln 0.7}{-0.000121} \approx 2948.$$

I.e., the fossilized leaf is about 2948 years old.