Section 4.1, Problem 16: $f(x) = \sqrt{x+1}$ and $g(x) = 3x$, so

(a) $(f \circ g)(4) = f(g(4)) = f(12) = \sqrt{13}$

(b) $(g \circ f)(2) = g(f(2)) = g(\sqrt{3}) = 3\sqrt{3}$

(c) $(f \circ f)(1) = f(f(1)) = f(\sqrt{2}) = \sqrt{\sqrt{2}+1}$

(d) $(g \circ g)(0) = g(g(0)) = g(0) = 0$.

Section 4.1, Problem 72: We are given that the demand equation for a firm's commodity

$$p = -\frac{1}{5}x + 200, \qquad 0 \le x \le 1000$$

and the firm's cost function

$$C = \frac{\sqrt{x}}{10} + 400,$$

where x is the number of items produces, $C(x)$ is the cost of producing x items and p is the price the firm sets for the commodity.

We are asked to find the cost C as a function of the price p. This is done in two steps.

First we solve the demand equation for x in terms of p:

$$p = -\frac{1}{5}x + 200 \quad \xrightarrow{\times 5} \quad 5p = -x + 1000 \quad \xrightarrow{+x-5p} \quad x = -5p + 1000.$$

Then we plug this expression for x into the cost function:

$$C = \frac{\sqrt{x}}{10} + 400 = \frac{\sqrt{-5p+1000}}{10} + 1000.$$

Section 4.2, Problem 62: We are asked to find the inverse function of $f(x) = \dfrac{4}{2-x}$. To do this, we first solve the equation

$$y = \frac{4}{2-x}$$

for the variable x in terms of y:

$$y = \frac{4}{2-x} \quad \xrightarrow{\text{flip both sides}} \quad \frac{1}{y} = \frac{2-x}{4} \quad \xrightarrow{\times 4} \quad \frac{4}{y} = 2 - x \quad \xrightarrow{+x-4/y} \quad x = 2 - \frac{4}{y}$$

Then we swap the roles of x and y and write $f^{-1}(x) = 2 - \dfrac{4}{x}$.

Check:

$$f^{-1}(f(x)) = 2 - \frac{4}{\frac{4}{2-x}} = 2 - \cancel{4} \cdot \frac{2-x}{\cancel{4}} = 2 - (2-x) = x \checkmark$$

and

$$f(f^{-1}(x)) = \frac{4}{2 - \left(2 - \frac{4}{x}\right)} = \frac{4}{4/x} = \cancel{4} \cdot \frac{x}{\cancel{4}} = x \checkmark$$

Section 4.3, Problem 106. The population $P(t)$ at time t in years of an endangered species is given by

$$P(t) = 30(1.149)^t.$$

The answers below are rounded to the nearest integer.

(a) The initial population is $P(0) = 30$.

(b) The population after 5 years will be $P(5) = 30(1.149)^5 \approx 60$.

(c) The population after 10 years will be $P(10) = 30(1.149)^{10} \approx 120$.

(d) The population after 15 years will be $P(15) = 30(1.149)^{15} \approx 241$.

(e) The population approximately *doubles* every 5 years (because $(1.149)^5 \approx 2$).