Homework 3 Solutions

Section 2.3, problem 28:

$$G(x) = 0 \implies (x+2)^2 - 1 = 0 \implies (x+2)^2 = 1 \implies x+2 = \pm\sqrt{1} = \pm 1$$
$$x+2 = -1 \implies x = -3 \quad \text{or} \quad x+2 = 1 \implies x = -1.$$

Conclusion: the zeros of $G(x) = (x+2)^2 - 1$ are x = -3 and x = -1 and these are also the x-intercepts of the function.

Section 2.3, problem 102: The area of the rectangular window is 306 and the dimensions of the window are x centimeters (width) and x + 1 centimeters (length). This leads to the equation

$$x(x+1) = 306 \implies x^2 + x - 306 = 0 \implies (x+18)(x-17) = 0 \implies x = 18 \text{ or } x = 17.$$

The negative solution (x = -18) doesn't make sense as a width, so the width of the window is x = 17 and its length is x + 1 = 18.

Section 2.4, problem 72: Find the quadratic function f(x) by using the form

$$f(x) = a(x-h)^2 + k$$

where (h, k) is the vertex. Now use the data to solve for a, h and k.

(i) We know that the vertex in this case is (1, 4), so h = 1 and k = 4. This means that

$$f(x) = a(x-1)^2 + 4,$$

and it remains to find a.

(ii) We also know that the graph passes through the point (-1, -8), and this means that f(-1) = -8 and therefore

$$a(-1-1)^2 + 4 = -8 \implies 4a + 4 = -8 \implies 4a = -12 \implies \boxed{a = -3}.$$

Conclusion: $f(x) = -3(x-1)^2 + 4 = -3(x^2 - 2x + 1) + 4 = -3x^2 + 6x + 1.$

Section 2.6, problem 10: Label the width of the rectangle with x and the length of the rectangle by y. If we think of y as the length of the side parallel to the river and x the length of the side perpendicular to the river, then the total amount of fencing is 2x + y and therefore

$$2x + y = 2000 \implies y = 2000 - 2x.$$

The area of the rectangular field is xy, and since y = 2000 - 2x we can express this as a function of x alone:

$$A(x) = x(2000 - 2x) = -2x^2 + 2000x.$$

The maximum value of the area occurs at the vertex of this quadratic function

$$x_v = -\frac{b}{2a} = -\frac{2000}{-4} = 500.$$

In words, the area is maximized when the width is x = 500 meters and the length is $y = 2000 - 2 \cdot 500 = 1000$ meters. The maximum area is $500 \cdot 1000 = 500,000$ square meters.