Chapter 26, Review Problem 1:

(a) **True:** 'P-value' and 'observed significance level' are two ways of saying the same thing.

(b) **False:** The null hypothesis is the claim that the results are due to chance variation. The alternative hypothesis says that the results are ***not*** due to chance.

Chapter 26, Review Problem 6:

(a) The sample percentage of women should be close to the population percentage of women, which is over 50%, so the observed sample percentage $102/350) \times 100\% \approx 29.1\%$ is ***extremely unlikely***. To attach an actual number to this, we can use the normal approximation.

We don't know the exact percentage of women in the population, but it is over 50%, so using 50% will make our estimate *conservative* — the probability we compute will be bigger than it should be.

The standard error is $SE_{\%} = (\sqrt{0.5 \times 0.5}/\sqrt{350}) \times 100\% \approx 2.67\%$, and the z-score corresponding to these statistics is

$$z = \frac{29.1\% - 50\%}{2.67\%} \approx -7.82,$$

so the probability of drawing 102 women (or fewer) in a random sample of 350 is approximately equal to the area under the normal curve to the left of $z = -7.82$. This probability is essentially 0, as we guessed.

(b) If 100 people are selected at random without replacement from a group of 102 women and 248 men, then the expected number of women is

$$\frac{102}{350} \times 100 \approx 29.$$

The standard error for the number of women in the sample is

$$SE = \sqrt{\frac{350 - 100}{349}} \times \sqrt{100} \cdot \sqrt{(102/350) \times (248/350)} \approx 3.85,$$

using the correction factor because 100 is a substantial proportion of 350 (without the correction factor, you would find that $SE \approx 4.55$). The z-score for a sample number of 9 women or fewer is

$$z = \frac{9 - 29}{3.85} \approx -5.19,$$

so the probability that a simple random sample of 100 people from this group includes 9 women or fewer is approximately equal to the area under the normal curve to the left of $z = -5.19$, which is about $p \approx 0\%$.

Technically, more possible than (a), but still extremely unlikely.

(c) Both the clerk and the judge were trying to avoid having women on the jury.

Chapter 27, Review Problem 3:

(a) This requires a two-sample z-test because we are comparing percentages from two different boxes: the 2000 box and the 2005 box.

(b) (*) Box model: There are two 0-1 boxes, one for the year 2000 and one for the year 2005. In both boxes there is a $\boxed{1}$ for every person in the population (that year) who rates clergy as 'high' or 'very high' and a $\boxed{0}$ for every other person. The surveys are like simple random samples from each of these two boxes.

(*) H_0: $p_{2000} \times 100\% - p_{2005} \times 100\% = 0 \quad H_1 : p_{2000} \times 100\% - p_{2005} \times 100\% > 0$.

This is a two-box test and p_{2000} and p_{2005} are the box *proportions* of $\boxed{1}$s in 2000 and 2005, respectively. We use a one-sided test here because we are testing to see if sex scandals reduced the proportion of people who rate the clergy highly. The null hypothesis says that the percentage of people who rate clergy highly has not changed — the (observed) difference in sample percentages is due to chance. The alternative hypothesis says that the observed difference is due to a difference in the boxes.

(c) $SE_{2000} = \dfrac{\sqrt{0.6 \times 0.4}}{\sqrt{1000}} \times 100\% \approx 1.55\%$, $SE_{2005} = \dfrac{\sqrt{0.54 \times 0.46}}{\sqrt{1000}} \times 100\% \approx 1.58\%$ and

$$SE_{\text{diff}} = \sqrt{SE_{2000}^2 + SE_{2005}^2} \approx 2.21\%.$$

The z-score is

$$z = \frac{60\% - 54\%}{2.21\%} \approx 2.7,$$

and the P-value is approximately equal to the area under the normal curve to the right of $z = 2.7$, which is $p \approx 0.35\%$.

Conclusion: The difference is (almost certainly) ***not*** due to chance. On the other hand, we ***cannot*** tell from the percentage data what did cause the percentage to go down — whether it was sex scandals or something else.

Comment: A two-sided test could also be used here, and the conclusions would be the same. I.e., if the investigators did not have an *a priori* expectation that support for the clergy had gone down, they would (should) have used a two-sided alternative. This would have doubled the P-value from $p \approx 0.35\%$ to $p \approx 0.7\%$, which is still tiny.