In Case Study 1 we used (hypothetical) data to find the (approximate) linear demand function for a gas station

$$q = -1.93p + 12.8,$$

where q = daily demand for gas (measured in 100s of gallons) and p = price per gallon (measured in dollars). In this Case Study, we continue with the same example try to answer the question: *what price should the gas station owner set to **maximize** her daily revenue and what will the maximum revenue be*?

The gas station's *daily revenue* is the amount of money they take in each day.† This revenue is equal to the price/gallon times the number of gallons sold at that price. Since we are measuring daily demand (q) in 100s of gallons, the number of gallons of gas sold per day is $100q$, and the gas station's daily revenue function is

$$r(p) = p \cdot (100q) = 100p(-1.93p + 12.8) = -193p^2 + 1280p.$$

This is a *quadratic* function whose graph appears below. Since the leading coefficient is negative, we know that $r(p)$ will have an absolute maximum value that occurs at the *vertex*. We also know that the p-coordinate of the vertex occurs halfway between the two zeros of $r(p)$.

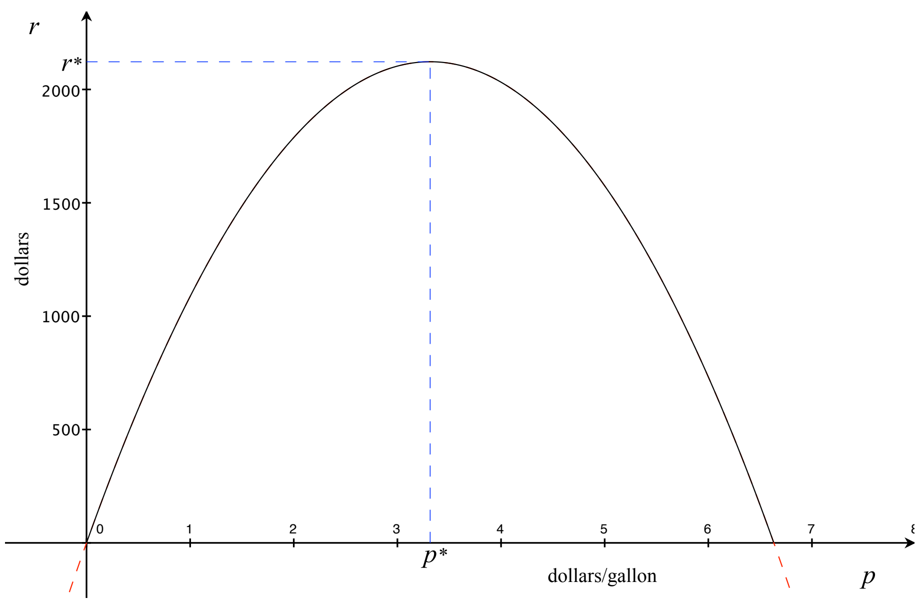

Figure 1: Graph of the revenue function $r = -193p^2 + 1280p$.

The zeros of $r(p)$ are the solutions of the equation $r = 0$, which can find (easily) by factoring in this case: $r = -193p^2 + 12800p = p(-193p + 1280)$. This means that $r = 0$ when $p = 0$ or when $-193p + 1280 = 0$, so the zeros are $p_1 = 0$ and $p_2 = 1280/193 \approx 6.632$. The halfway point between p_1 and p_2 is $p^* = 3.316$ — this is the p-coordinate of the vertex, so this is the revenue-maximizing price. Finally, the maximum (possible) daily revenue is $r^* = r(p^*) \approx 2122.28$.

†We are focusing here only on revenue from gas sales.