

In Economics a **demand function** is a function that relates the demand for a firm’s good (i.e., how much of that good consumers will buy per unit time) to other variables. The variable most directly related to the demand for a good is the price that the firm sets for its product. The relevant economic theory is *the law of demand* that states that (for most goods), the demand  $q$  (for *quantity* demanded) for a good will be a **decreasing** function of the price  $p$  of that good.<sup>†</sup> The higher the price, the lower the demand and *vice versa*.

On to the case study...

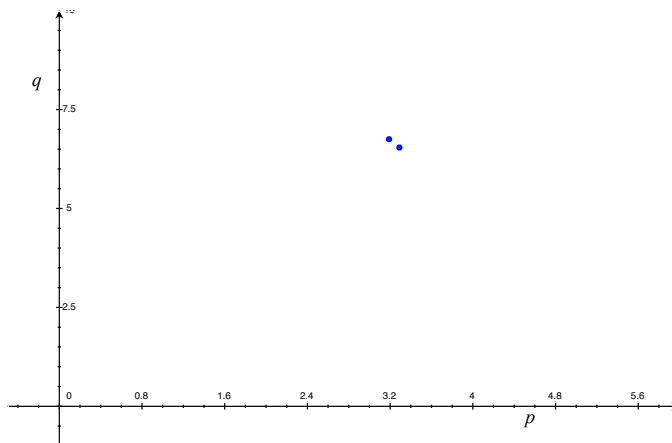
The owner of the ACME gas station wants to find a function that describes the amount of (regular, 87 octane) gas she will sell in a day (measured in 100s of gallons), as it depends on the price-per-gallon (measured in dollars) that she sets.

**First attempt — two data points**

The owner observes that when the price/gallon is \$3.19, the daily demand is about 675 gallons on average and when the price/gallon is \$3.29, the daily demand is about 654 gallons on average. These two observations give her two pairs of values for her variables:

$$(p_1, q_1) = (3.19, 6.75) \quad \text{and} \quad (p_2, q_2) = (3.29, 6.54),$$

remembering that  $q$  is measured in 100s of gallons, so 675 gallons translates to  $q = 6.75$ , for example. The figure below shows these two points (as blue dots) in the  $(p, q)$ -coordinate system.



The owner wants a *linear* model for this demand function, and knows that given a pair of points there is exactly one line that passes through them, and she can find its equation by (a) finding its slope using both points and then (b) using the point-slope formula.

The slope of the line that passes through the points  $(3.19, 6.75)$  and  $(3.29, 6.54)$  is

$$m = \frac{q_2 - q_1}{p_2 - p_1} = \frac{6.54 - 6.75}{3.29 - 3.19} = -2.1.$$

Next, she uses the point-slope formula (with the first point) to find

$$q - q_1 = m(p - p_1) \implies q - 6.75 = -2.1(p - 3.19).$$

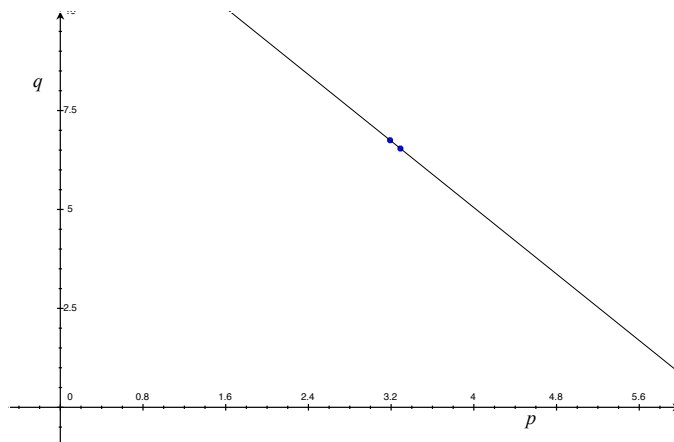
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<sup>†</sup>The law of demand applies to what are called *normal* goods. In contrast, a *Veblen* good is one for which the demand rises as the price goes up, for example luxury items that appeal to consumers because they are exclusive, where the exclusivity is in part due to their high price.

Simplifying, she finds that the demand function for her gas station is

$$q = -2.1p + 13.449,$$

where  $p$  is the price per gallon (in dollars) and  $q$  is the quantity of gas she sells per day at that price (in hundreds of gallons). The graph of this demand function appears in the figure below, and we can see that it is indeed decreasing, as expected.



Now that we have a (linear) model for the demand for gas, we see what we can learn from it. For example, the owner can predict what the daily demand will be at other price-points. E.g., if the price is lowered to  $p = 2.99$ , her model says that she can expect the demand to increase to

$$q = -2.1 \cdot 2.99 + 13.449 = 7.17,$$

or 717 gallons per day. If on the other hand, she raises the price to  $p = 3.39$ , then she will only sell about

$$q = -2.1 \cdot 3.39 + 13.449 = 6.33$$

or 633 gallons a day.

In linear models, the slope coefficient has an important interpretation. Since the slope of a line is the (constant) rate of change along the line, we see that

$$-2.1 = \frac{\Delta q}{\Delta p} \implies \Delta q = -2.1\Delta p.$$

So if  $\Delta p = 1$ , i.e., if the price goes up by one dollar, then the demand for gas will go down by 2.1 or 210 gallons per day. More practically speaking, if  $\Delta p = 0.1$ , then  $\Delta q = -.21$ , which we can interpret to say that the daily demand for gas will decrease by 21 gallons for every 10 cent increase in the price.

### Second attempt - more data

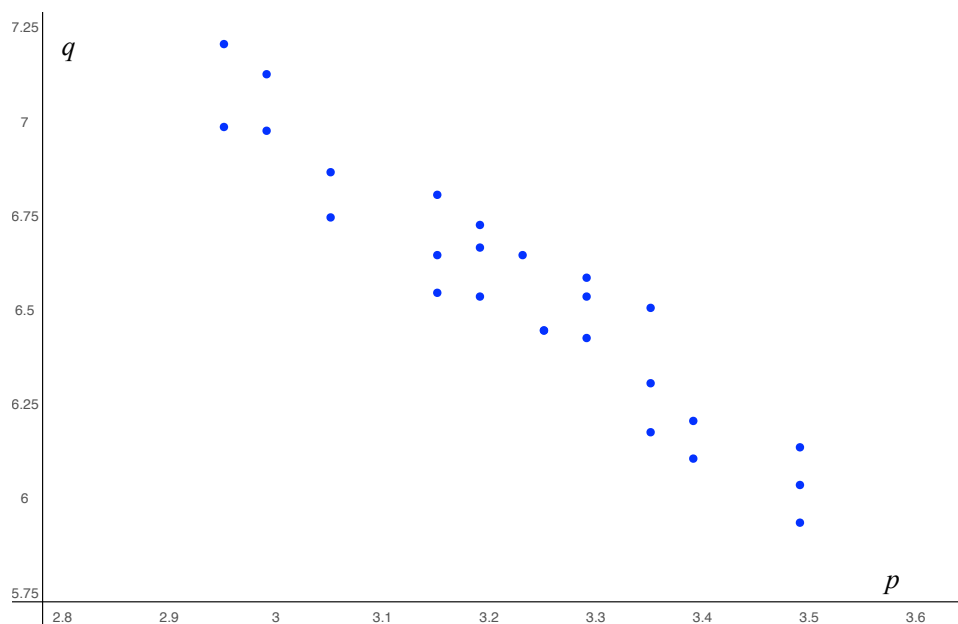
Our first attempt at finding a demand function for the ACME Gas Station has one glaring weakness. It only uses two data points, and therefore leaves out a lot of information. Moreover, the two data points themselves were summaries of the data, since they reported *average* demands for the given prices.

To get a better picture of how the price of gas is related to the demand for gas at her gas station, ACME's owner collected data over several weeks. The data is summarized in the table

that follows.

Price (dollars)	Demand (100s of gallons)	Price (dollars)	Demand (100s of gallons)
3.19	6.73	3.05	6.87
3.15	6.81	3.05	6.75
3.23	6.65	3.15	6.55
3.29	6.54	3.15	6.65
3.29	6.59	2.99	6.98
3.19	6.67	2.99	7.13
3.19	6.54	3.29	6.43
2.95	7.21	3.25	6.45
2.95	6.99	3.25	6.45
3.49	5.94	3.35	6.31
3.39	6.11	3.35	6.51
3.39	6.21	3.49	6.04
3.35	6.18	3.49	6.14

The same data is plotted in the *scatter plot* below.



A quick look at the data tells us that there is can be no function  $q = f(p)$  that precisely models this data because the points in the scatter plot don't pass the *vertical line test*. We can see this in the table too, where there are several prices for which there are different levels of demand on different days.

What does this tell us about the relation between price and demand (for gas)? It tells us that (daily) demand depends on more than just the price. There are other variables in play whose effect is seen when the same price (on different days) result in different levels of demand.

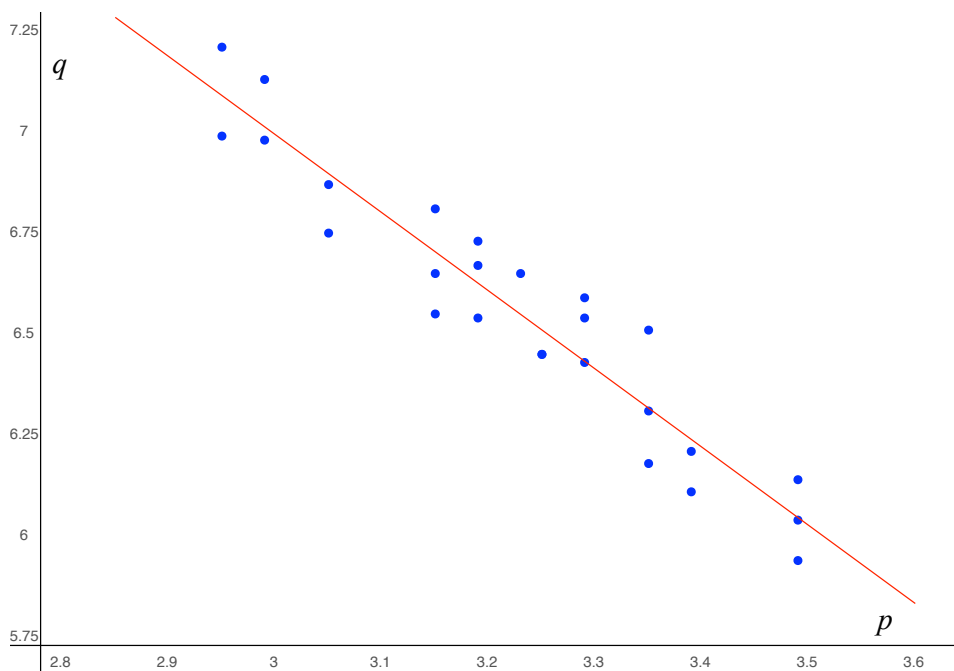
In spite of the fact that demand is not a function of price alone, we can still try to *approximate* the relation between these two variables with a functional relation. To do this, we observe that the scatter plot on the previous page has a linear look to it — the points appear to be going down from left to right at a fairly constant rate.

This means that we can try to find a linear function that *approximates* the relation between price and demand in the data. Geometrically speaking, we look for a straight line that *best fits* the points in the scatter plot. Loosely speaking, the *line of best fit* for a set of data, is the line that comes as close as possible to all the points in the data. The slope and intercept coefficients of the *line of best fit* are computed from the coordinates of data — the formulas can be a little long, depending on how much data there — and we usually use software to calculate them.

For the price and demand data for gas from the table above, I used the MacOS grapher utility to plot the line of best fit and to find its equation.<sup>‡</sup> This utility returned the linear function

$$q = -1.93p + 12.8$$

as the model for predicting demand for gas from the price. As with the simpler model we calculated before, we interpret the slope coefficient as the rate of change of demand with respect to price. Concretely, for every 10 cent increase in price, demand for gas will decrease by about 19.3 gallons, on average. The Grapher utility was also kind enough to plot the line of best fit, in red below.




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<sup>‡</sup>You can also use Wolfram Alpha to do this. In the input line, type *linear fit* followed by the set of data (pairs of points)  $\{\{x_1, y_1\}, \{x_2, y_2\}, \dots, \{x_n, y_n\}\}$ , then hit return.