AMS 3 — WINTER 2018

Midterm 2 Review

1. Find the polynomial of smallest degree whose graph matches the one in Figure 2, below.

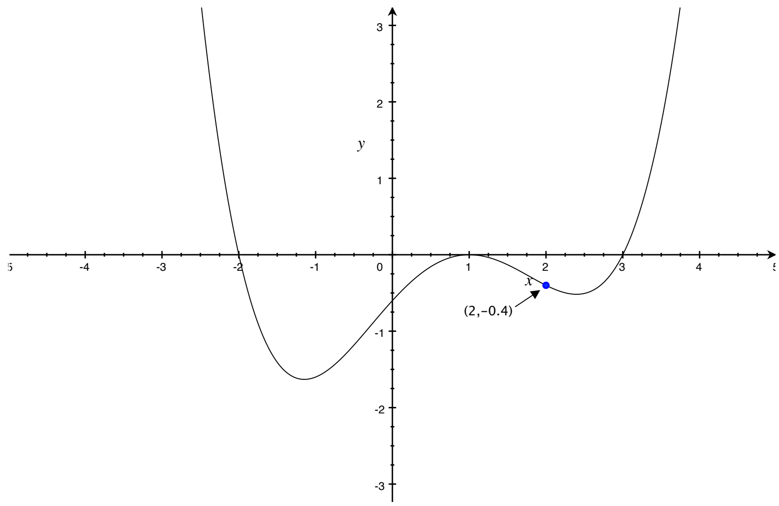

Figure 1: Graph for problem 2.

2. Follow steps 1 through 5 on page 205 of the textbook to analyze the polynomial

$$f(x) = 0.5(x+1)(x-2)^3.$$

3. Find the *rational* zeros of the polynomial

$$p(x) = 6x^3 - 17x^2 - 5x + 6.$$

4. Consider the rational function:

$$R(x) = \frac{x^2 - x - 2}{2x^2 + 5x - 3}.$$

(a) Factor the numerator and denominator of $R(x)$. What is the domain of this function?

(b) Find the x and y intercepts of $R(x)$ and determine the behavior of the function at each x intercept ('crossing' or 'touching').

(c) Find the vertical asymptotes of $R(x)$ and determine the behavior of $R(x)$ on either side of each vertical asymptote.

(d) Find the horizontal or oblique asymptote of $R(x)$, or say why no such line exists. If there is a horizontal/oblique asymptote, find any point(s) of intersection of the graph of $R(x)$ and the asymptote.

(e) Find the intervals where $R(x) > 0$ and $R(x) < 0$.

(f) Use all the information you have found to sketch the graph of $R(x)$.

5. For the functions

$$f(x) = \frac{1}{x+2} \quad \text{and} \quad g(x) = x^2 - 1$$

find the composite functions

$$f \circ f, \quad f \circ g, \quad g \circ f \quad \text{and} \quad g \circ g$$

and find the domain of each one.

6. Find the inverse functions of $H(x) = \dfrac{x+1}{2x+3}$ and $G(x) = 4x^3 - 1$. Find the domains of both inverses.

7. The surface area of an inflatable globe of radius r is

$$S = 4\pi r^2.$$

The radius of the ball increases as a function of time t (in seconds), according to the rule $r = 2t^{1/3}$. Find the surface area of the ball as a function of t.

8. According to U.S. Census bureau, the world's (human) population in 2013 was 7.13 billion and was growing at a rate of 1.1% per year. E.g., in 2014, the population was 1.1% greater than in 2013, so it was

$$7.13 + (1.1\%) \cdot 7.13 = 7.13 \cdot 1.011 \approx 7.208 \text{ billion.}$$

(a) Assuming that the world maintains this rate of growth, find the function

$$P(t) = \text{ world's population, in billions, } t \text{ years after 2013.}$$

(b) According to the model you found in (a), what will the world's population be in 2050? Round your answer to the nearest million.

(c) According to this model, in what year will the world's population reach 20 billion?

9. Newton's law of cooling/heating (see section 4.8) states that the temperature $u(t)$ at time t of a body immersed in a medium of constant ambient temperature T, can be modeled by the function

$$u(t) = T + (u_0 - T)e^{kt},$$

where u_0 is the initial temperature of the body (at the time of immersion) and $k < 0$ is a constant related to the heat conduction properties of the body.

A metal ball is heated to 500° Celsius and then immersed in a vat of ice water, that is kept at 0° Celsius. After 10 minutes the temperature of the ball is 400° Celsius. When will the ball reach a temperature of 100° Celsius?

10. A video spreads virally on a certain social media platform according to the model

$$N(t) = \frac{1,000,000}{1 + be^{-kt}},$$

(the logistic model), where $N(t)$ is the number of people who have seen the video, and the parameters b and k are (currently) unknown. At 1:00 am on Monday, 1000 people have seen the video and by 10:00 am on that same Monday, $10,000$ people have seen video.

(a) Find b and k. Round k to two decimal places.

(b) How many people will have seen the video by 11:00 pm on that same day?

(c) At what time (and day) will $900,000$ people have seen the video?

11. The wooden handle of a primitive axe found at an archaeological dig contains 30% of its initial amount of carbon-14. How old is the axe handle? You may assume that the half life of carbon-14 is 5730 years.