1. Consider the function $k(x) = 3\sin\left(\frac{\pi}{4}(x-2)\right) + 1$. What is the period, amplitude and average value of $k(x)$? What are the maximum and minimum values of $k(x)$ and at which points do the minimum and maximum values occur? Use this information to sketch a graph of the function.

2. *Constructing a window.* A window is in the shape of a rectangle with a semicircular top, as in Figure 1, below. The perimeter of the window is to be 10 feet — what should the dimensions of the window be to maximize its area?

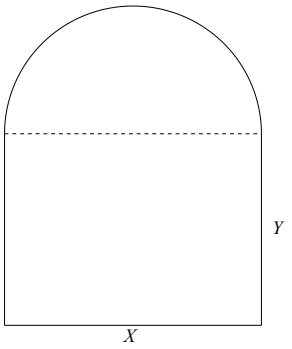

Figure 1: Window for problem 1.

3. Consider the rational function:

$$R(x) = \frac{x^3 + x^2 - 9x - 9}{2x^2 - 4x - 16}.$$

(a) Factor the numerator and denominator of $R(x)$. (Hint: $x^3 + x^2 = x^2(x+1)$).

(b) Find the x and y intercepts of $R(x)$ and determine the behavior of the function at each x intercept ('crossing' or 'touching').

(c) Find the vertical asymptotes of $R(x)$.

(d) Find the intervals where $R(x) > 0$ and $R(x) < 0$. Use this information to determine the behavior of $R(x)$ on either side of the vertical asymptotes.

(e) Find the horizontal or oblique asymptote of $R(x)$. Find any point(s) of intersection of the graph of $R(x)$ and the asymptote. Describe the behavior of $R(x)$ as $x \to \infty$ and $x \to -\infty$

(f) Use all the information you have found to sketch the graph of $R(x)$. Clearly mark intercepts and any other points of interest (e.g., the points you used to determine signs).

4. Find the inverse functions of

$$f(x) = e^{3x-1} + 1,\ g(x) = \log_2(x^3+1) \text{ and } h(x) = \sqrt[3]{4x-1} + 2$$

and find their domains.

5. Sketch the graph of $h^{-1}(x)$ from problem 4. Indicate the x and y intercepts.

6. Simplify the following expressions, as described.

(a) Express

$$y = \frac{1}{3}\log_4(x^2+1) - 3\log_2(x-4)$$

as a single logarithm ***base* 2**. I.e., $y = \log_2(F(x))$.

(b) Express

$$w = \log\left(\frac{(x^3+5)^2(x-7)^3}{\sqrt[4]{2x+5}}\right)$$

as a sum of multiples of logarithms. I.e., $w = a\log(f(x)) + b\log(g(x)) + \cdots$, where the constants $a, b, \ldots$ can be positive or negative.

7. The tides at Santa Cruz beaches exhibit periodic behavior. At high tide, which occurs at 8 am, the height of the water is 11.5 meters. At the subsequent low tide, which occurs at 2 pm, the height of the water is 8.5 meters.

(a) Find the *period, amplitude* and *average height* of the tides.

(b) Find the function $H(t) = b + A\cos(\omega(t-t_0))$, that gives the height (in meters) of the water at time t (in hours after midnight).

(c) What is the height of the water at 10 am? Do not use a calculator to find the answer.

8. The population of Tribbles on the Starship Enterprise is growing at a rate of 25% an hour. At 7 am on Monday, there are 20 Tribbles.

(a) How many Tribbles will there be at 8 am on Monday.

(b) Find the function $T(t) = a \cdot b^t$ = number of Tribbles at time t (hours after 10 am on Monday). I.e., find a and b.

(c) Express $T(t)$ in the form $T(t) = \alpha e^{\beta t}$. I.e., find α and β.

(d) At what time on what day will the Tribble population reach 500? Round your answer to the nearest hour.

9. Joe borrows \$500,000 from Mike, and agrees to pay the full amount back, plus interest in 10 years. The agree on an interest rate of $r = 4.5\%$ compounded monthly. How much will Joe have to pay Mike at the end of 10 years?. How much interest did he pay?

10. Express the values of the following trigonometric functions as rational numbers, rational multiples of $\sqrt{2}$, $\sqrt{3}$ or $\sqrt{6}$ or sums/differences of numbers like that. (No calculators)

(a) $\sec(2\pi/3) =$

(b) $\csc(3\pi/4) =$

(c) $\tan(-2\pi/3) =$

(d) $\sin(7\pi/4) =$

(e) $\sin(\pi/12) =$

You will find the identity

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \sin\beta\cos\alpha,$$

useful for (e) after you express $\pi/12$ as a difference of two angles whose sin and cos values are known.

11. A fossilized insect contains 20% of its initial amount of carbon-14. How many years ago did the insect die? You may assume that the half life of carbon-14 is 5730 years.

12. Sketch the graphs of the following functions. For each one, mark the x and y intercepts, if they exist, and indicate the behavior of the function as $x \to \pm\infty$.

(a) $f(x) = 2\ln(x+3)$.

(b) $g(x) = 2^{x-3} - 5$

13. Solve the equation $4^{x+2} = 2 \cdot 5^{x-1}$. Express your answer in terms of the natural logarithm function. Don't use a calculator.

14. Solve the equation $7\sin^2 x + 3\cos^2 x = 6$. List all possible solutions.

(Hint: $\sin^2 x + \cos^2 x = 1$.)

15. Find the linear function $y = f(x)$ that passes through the points $(-1, 2)$ and $(2, 3)$.